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**Cross-lag Mean Structure Models vs
Latent Curve Structural Equation Models
of Change**

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OUTLINE

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Mean and Covariance Structures

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Mean/Covariance Growth Curve Models

A Study on the Gateway Hypothesis of Drug Use

A Hierarchy of Modeling Challenges

1. A single group covariance structure model

e.g., $\Sigma = \Lambda\Phi\Lambda' + \Psi$

2. A single group covariance structure model with fixed a priori loadings

e.g., simplex structure $\Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

e.g., circumplex structure $\Lambda = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

- e.g., initial status growth curve model

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \end{bmatrix}$$

3. Single Sample Mean Structure Model

- (a) $\mu = \mu(\theta)$, Σ free [note: may be easier than CSA]
- (b) $\mu = \mu(\theta)$, $\Sigma = \Sigma(\gamma)$ (no fixed nonzero parameters)
- (c) $\mu = \mu(\theta)$, $\Sigma = \Sigma(\theta)$ (no fixed nonzero parameters)
- (d) $\mu = \mu(\theta)$, $\Sigma = \Sigma(\theta)$ (many fixed nonzero parameters)

Example of (d): Standard Growth Curve Model

$$\mu = \Lambda\mu_{\xi}, \Sigma = \Lambda\Phi\Lambda' + \Psi, \Lambda \text{ all fixed parameters}$$

4. Multiple Sample Covariance Structures with cross-group constraints

5. Multiple Sample Mean and Covariance Structures with cross-group constraints

6. Multiple Sample Mean and Covariance Structures with cross-group constraints, including fixed nonzero parameters (e.g., growth curves in 2 or more samples)

7. Mixture variants of #6, that is, multiple mean and covariance structures based on unknown and probabilistic group membership

8. Now add a multilevel or hierarchical structure (e.g., Within and Between Cluster models) to each of these, etc.

Longitudinal Models as Covariance Structures Only

A comment on difference scores – factors may disappear

$$x_1 = \Lambda \xi_1 + \varepsilon_1$$

$$x_2 = \Lambda \xi_2 + \varepsilon_2 \text{ (Note same } \Lambda \text{)}$$

This means that the difference score has structure

$$x_2 - x_1 = (\Lambda \xi_2 + \varepsilon_2) - (\Lambda \xi_1 + \varepsilon_1)$$

$$= \Lambda(\xi_2 - \xi_1) + (\varepsilon_2 - \varepsilon_1)$$

So if the factor scores stay quite stable, the variance of the difference factor $(\xi_2 - \xi_1)$ may get quite small. Most of the variance in $(x_2 - x_1)$ may be due to $(\varepsilon_2 - \varepsilon_1)$.

An underused alternative is latent variable residual gain (see Newcomb, Scheier, & Bentler, 1993)

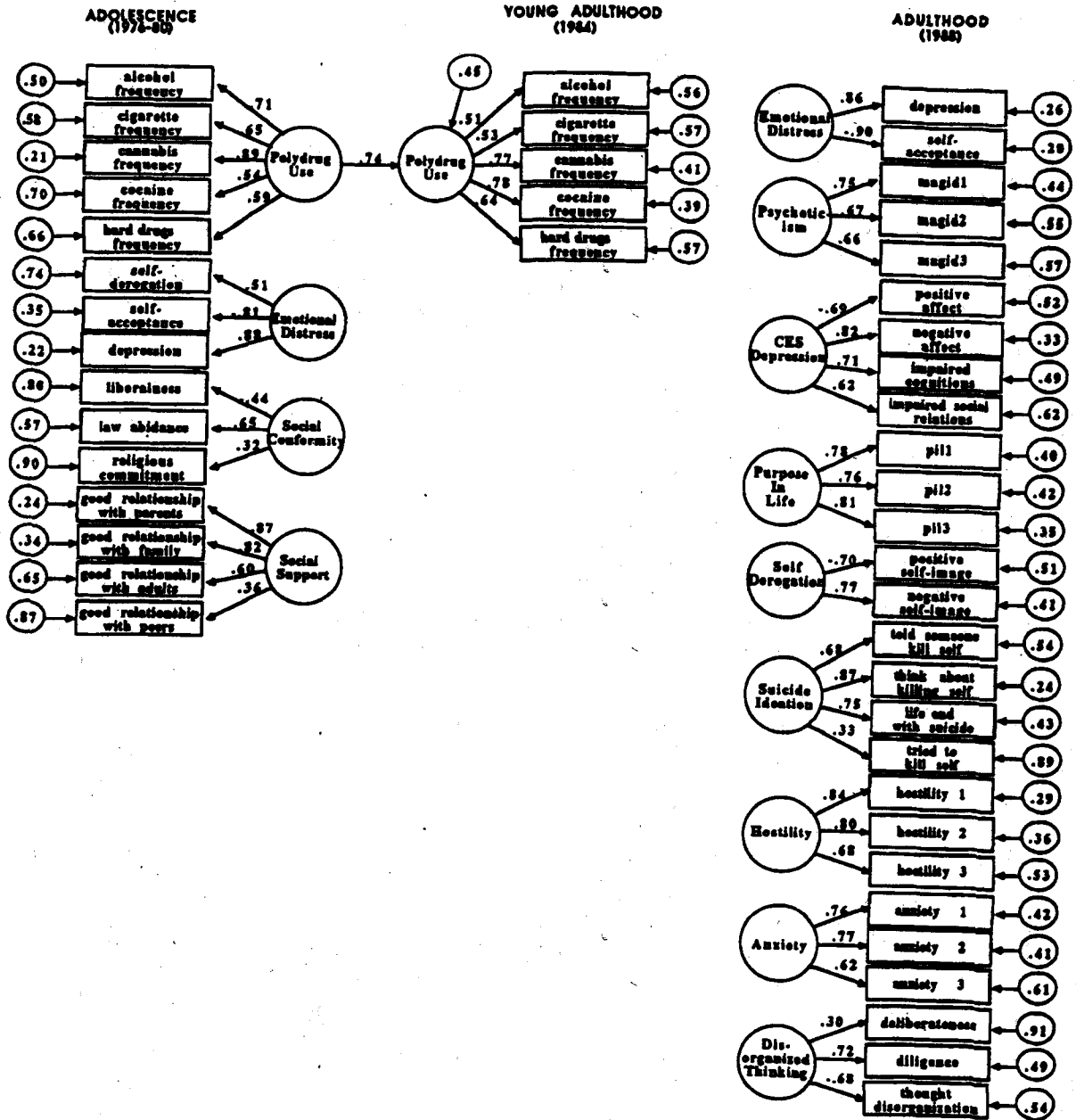
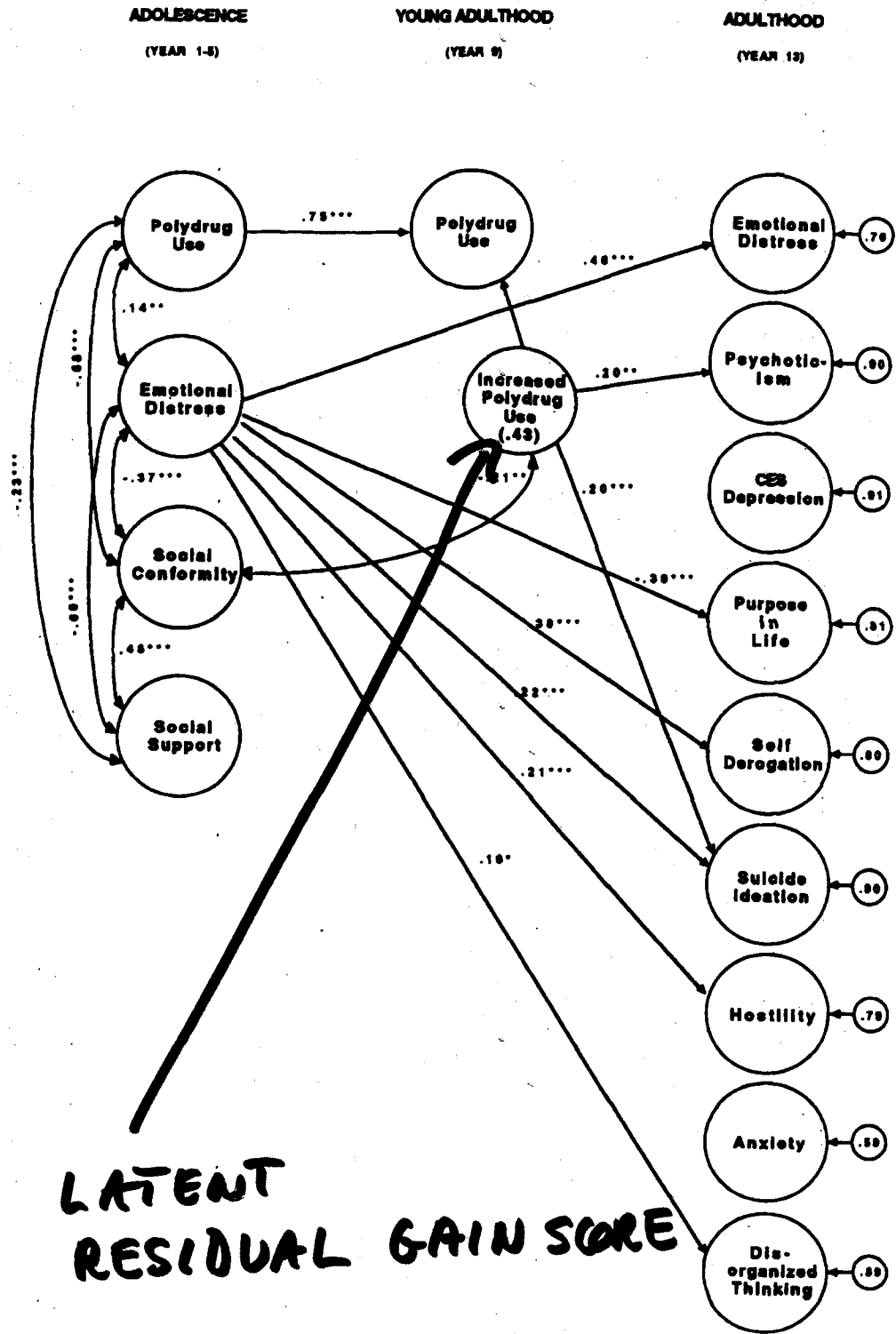


Figure 1. Final confirmatory factor analysis model. (Large circles represent latent constructs, rectangles are measured variables, and small circles with numbers are residual variables. Factor loadings are standardized and all are significant [$p < .001$]. Not depicted in the figure are two-headed arrows [i.e., correlations] joining each possible pair of factors. Estimates for these correlations are given in Table 3. CES = Center for Epidemiologic Studies; magid = magic ideation; pil = purpose in life.)

tors to the various adult mental health measures (both latent and observed). These paths are all from the final structural model and must be considered in concert with results from the same

model given in Figure 2 and Table 4. These specific or nonstandard paths provide a detailed picture of more specific influences of both drug and nondrug adolescent variables on adult mental health (see



LATENT RESIDUAL GAIN SCORE

$$RES(=D2) = POLY2(F2) - .75 \times POLY1(F1)$$

$$OR \quad POLY2(F2) = .75 \times POLY1(F1) + D2$$

WEIGHTED LATENT DIFFERENCE SCORE

Consequences of Adolescent Drug Use:

Impact on the Lives of Young Adults

by Michael D. Newcomb & Peter M. Bentler

(Newbury Park: Sage Publications, 1988)

Jr. High Students Followed to Adulthood

Natural Variation in Drug Use

Many Initial Control Variables

Latent Variable Methodology

Nonstandard (non-LISREL) models

Consequences on: FAMILY FORMATION AND STABIL-

ITY; CRIMINALITY AND DEVIANT BEHAVIOR; SEXUAL

BEHAVIOR AND INVOLVEMENT; EDUCATIONAL PUR-

SUITS; LIVELIHOOD PURSUITS; MENTAL HEALTH;

SOCIAL INTEGRATION

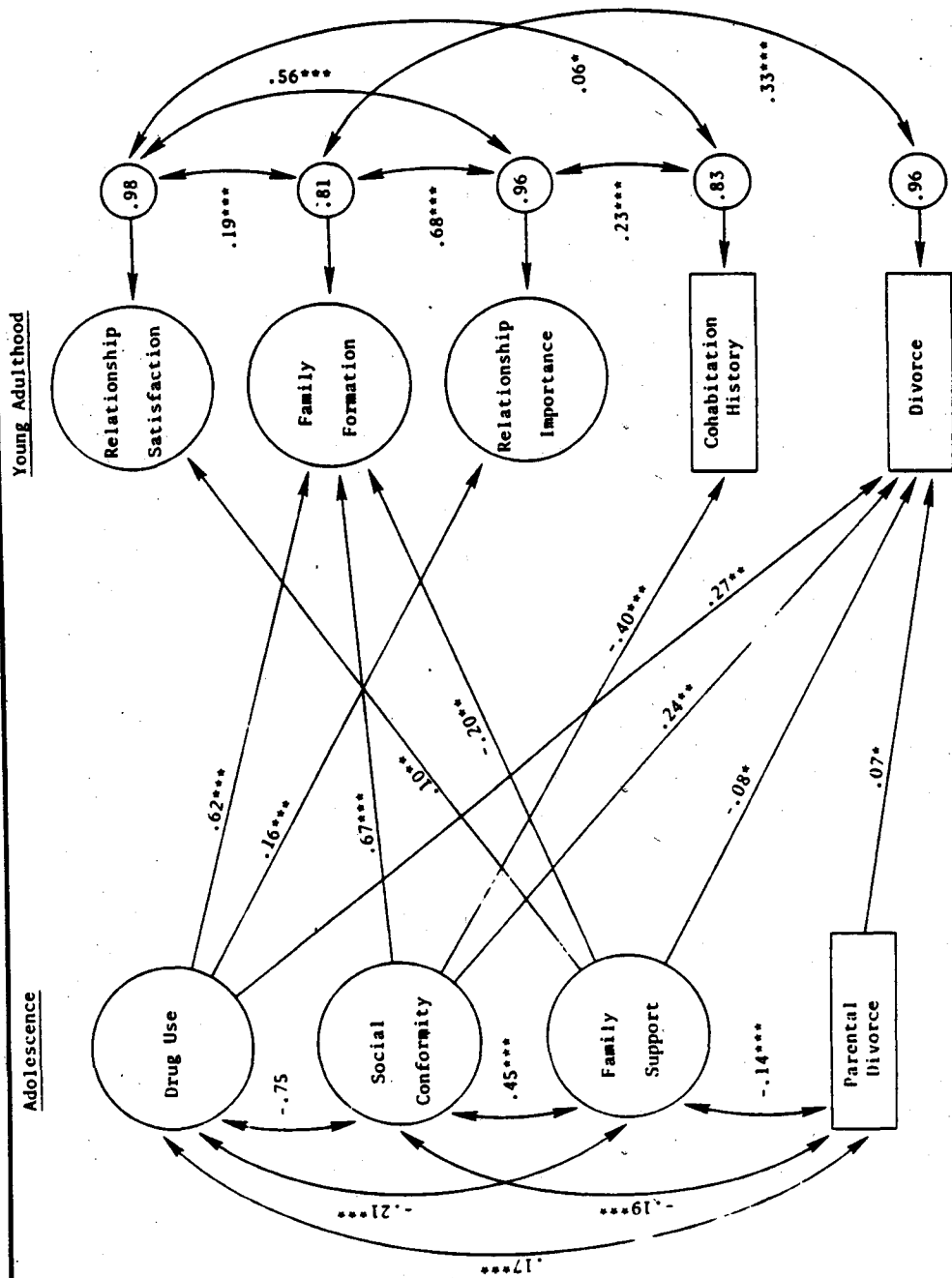


Figure 7.2. Final Structural or Path Model for the Family Formation Variables

NOTE: The measurement portion of this model is not depicted for simplicity. Two-headed arrows reflect correlations and single-headed arrows represent across-time regression effects. Parameter estimates are standardized, residual variables are variances, and significance levels were determined by critical ratios (* $p < .05$; ** $p < .01$; *** $p < .001$). Nonstandard effects for this model are given in Table 7.2.

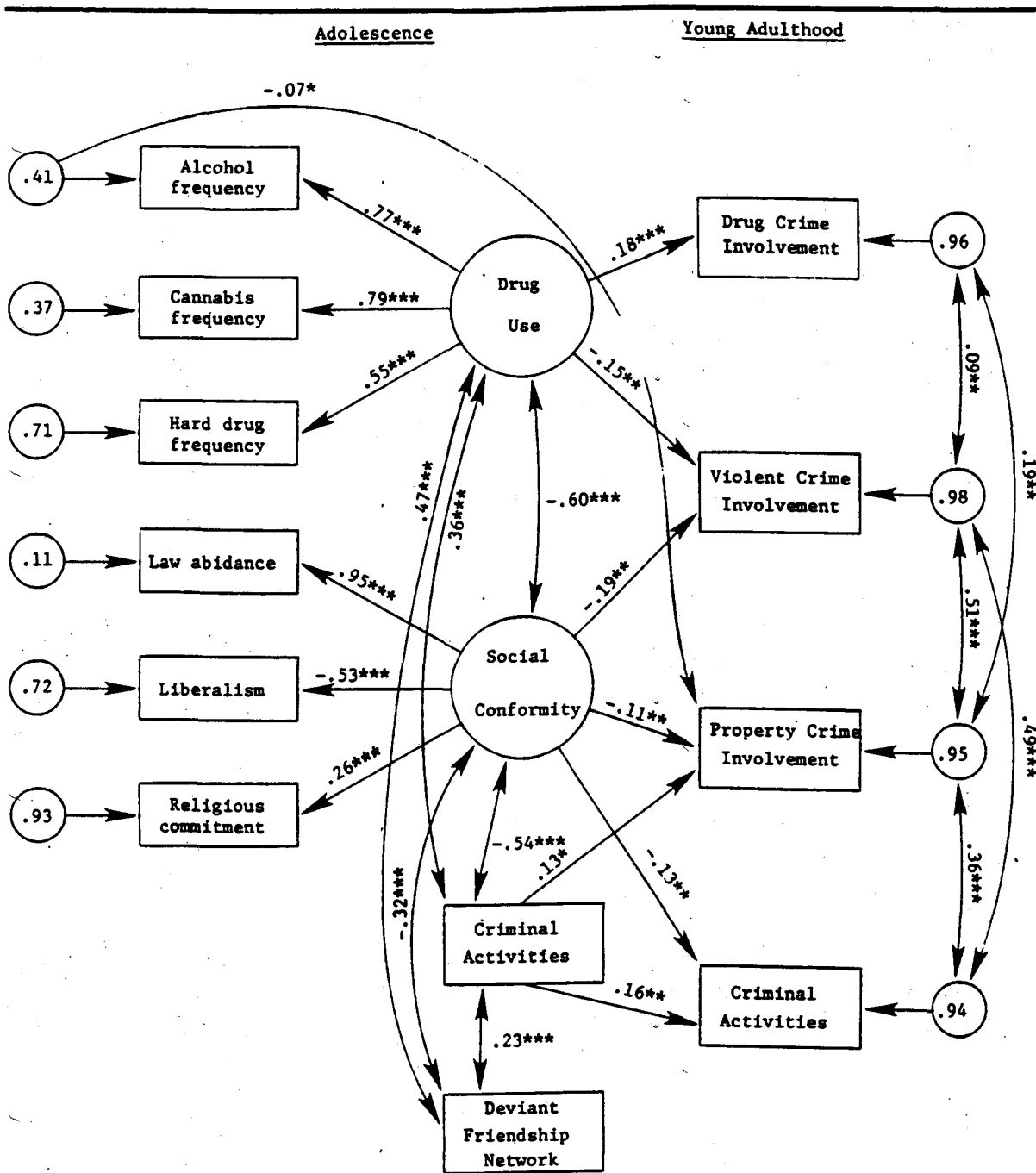


Figure 8.3. Final Structural Model Including Measurement Model for the Distribution-Free Estimates (AGLS) of the Deviant Behavior Variables

NOTE: Large circles reflect latent factors, rectangles represent measured variables, and small circles with numbers are residual variances. Two-headed arrows are correlations and single-headed arrows represent regression effects. Parameter estimates are standardized and significance levels were determined by critical ratios (*p < .05; **p < .01; ***p < .001).

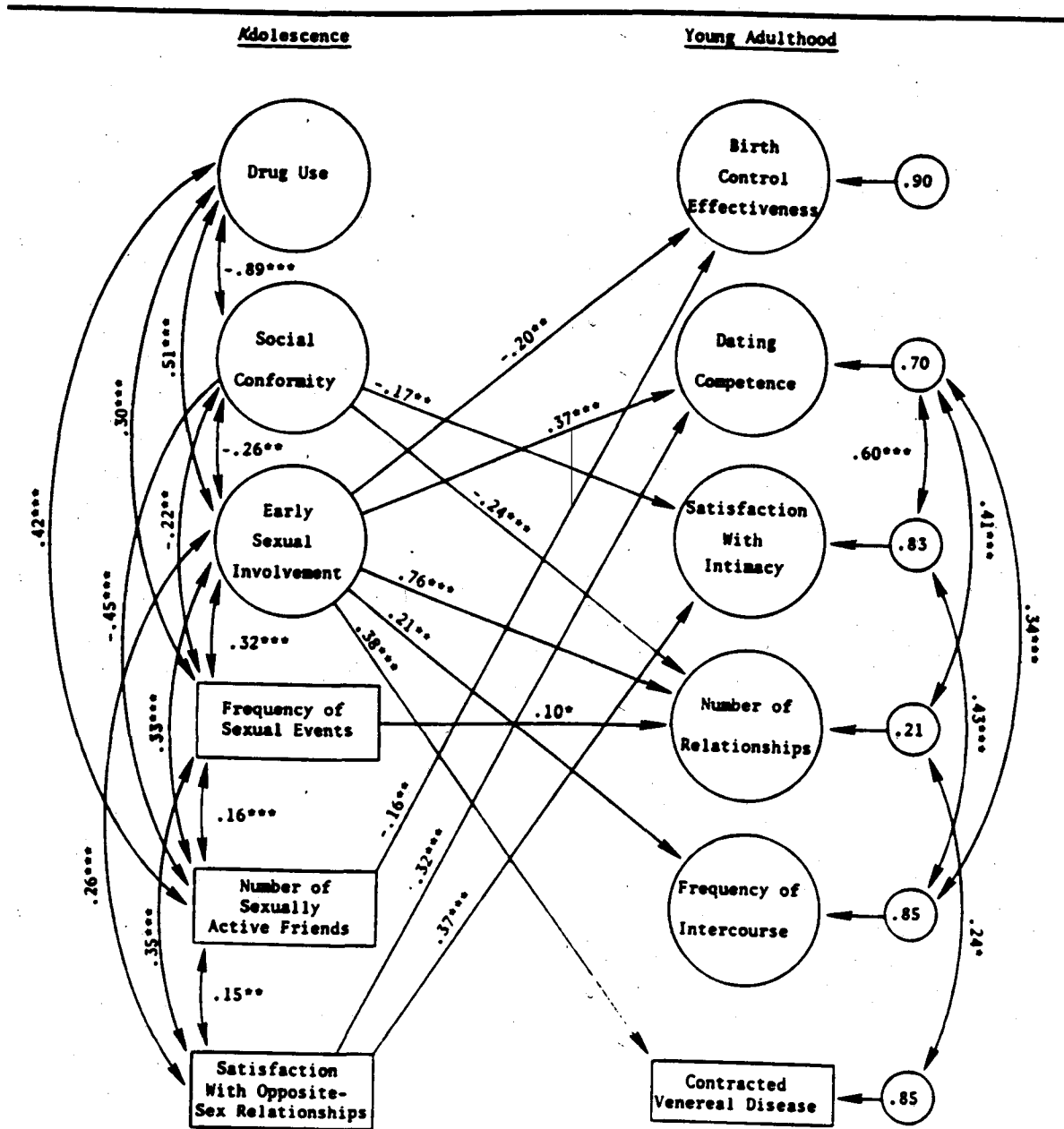
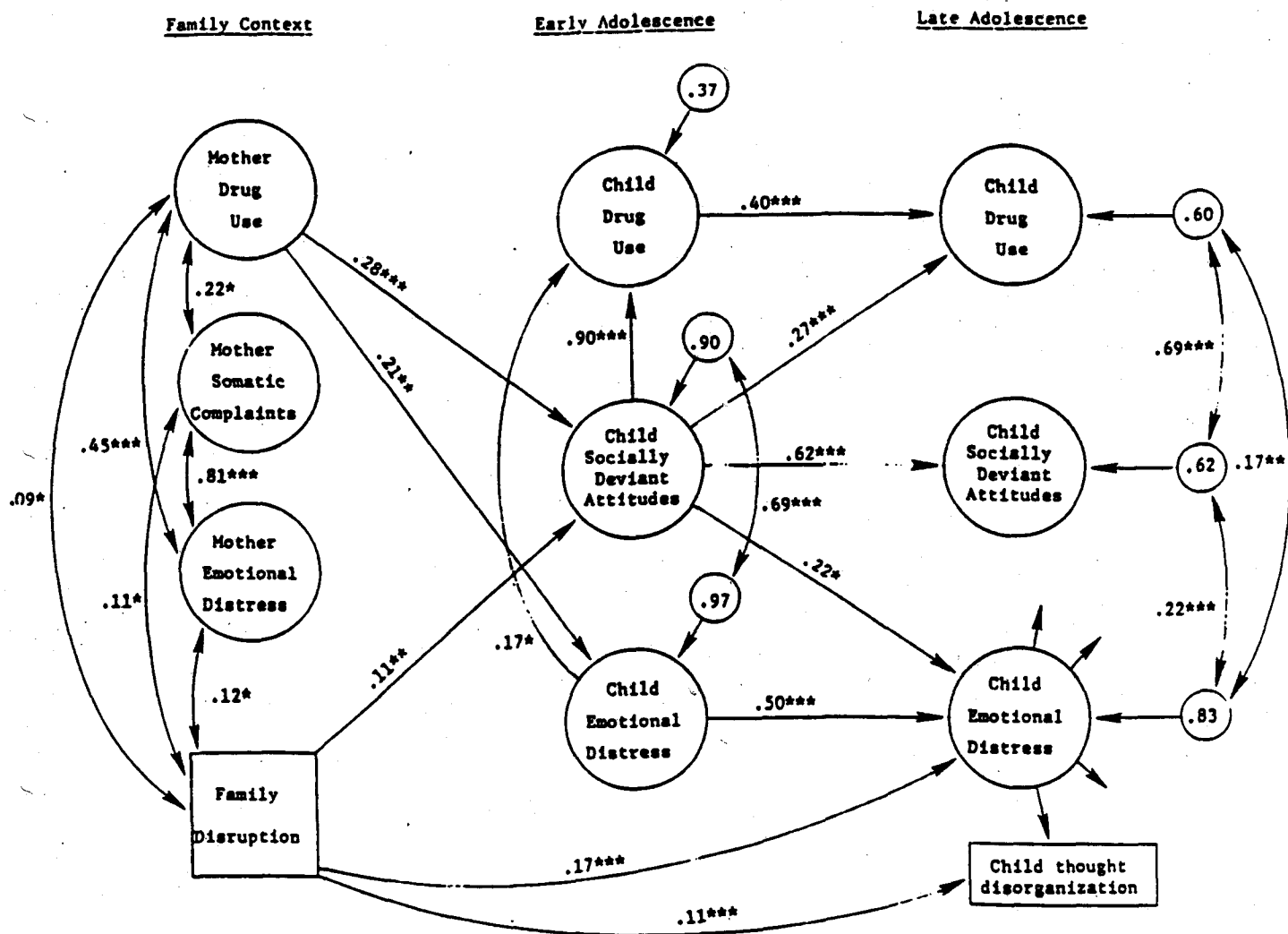


Figure 9.4. Final Structural or Path Model for the Men's Sexual Behavior Variables

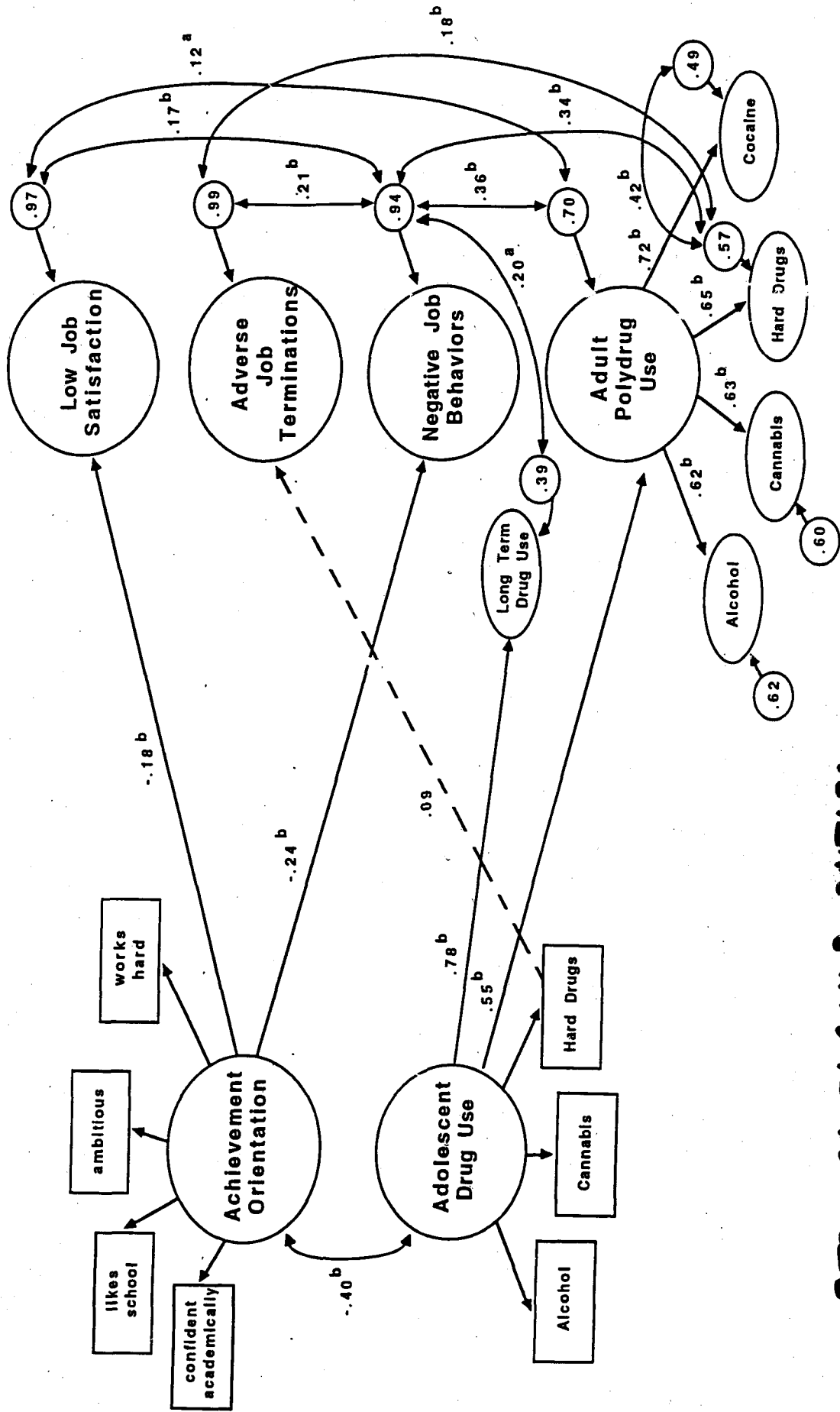
NOTE: The measurement portion of this model is not depicted for simplicity. Two-headed arrows reflect correlations and single-headed arrows represent across-time regression effects. Parameter estimates are standardized, residual variances are variances, and significance levels were determined by critical ratios (* $p < .05$; ** $p < .01$; *** $p < .001$). Nonstandard effects for this model are given in Table 9.4.



Newcomb, M. D., & Bentler, P. M. (1988). The impact of family context, deviant attitudes, and emotional distress on adolescent drug use: Longitudinal latent-variable analyses of mothers and their children. *Journal of Research in Personality*, 22, 154-176.

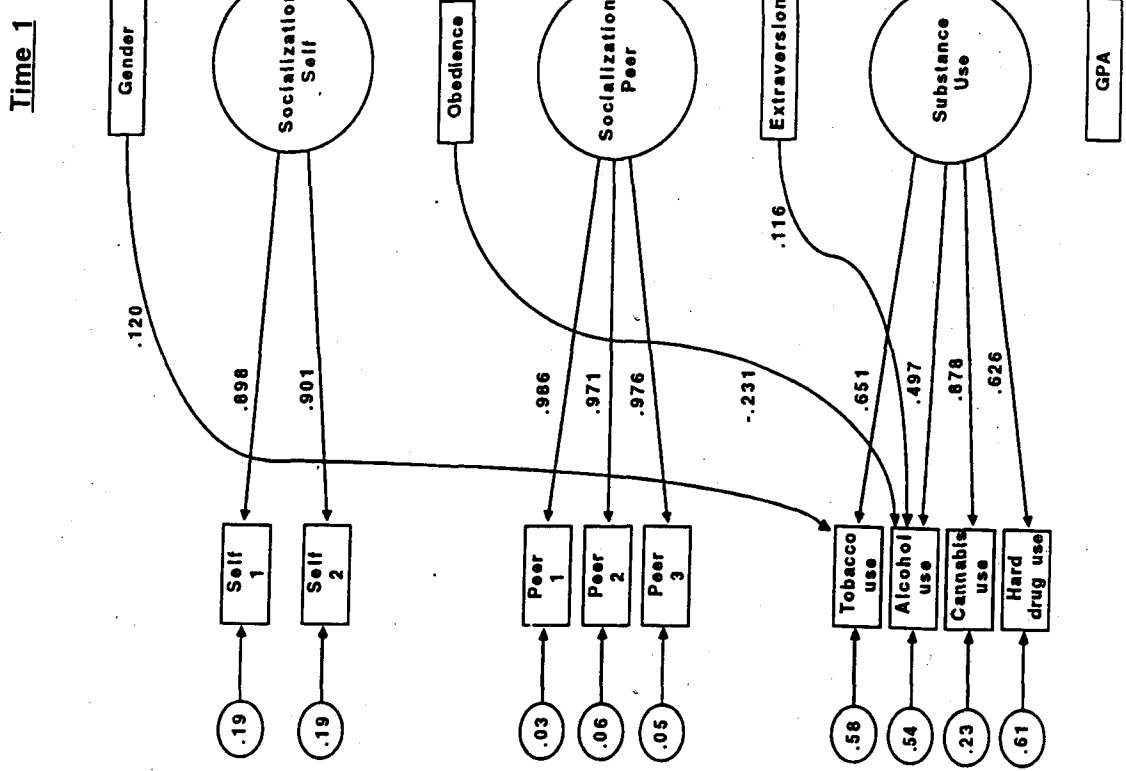
Adolescence

Young Adulthood

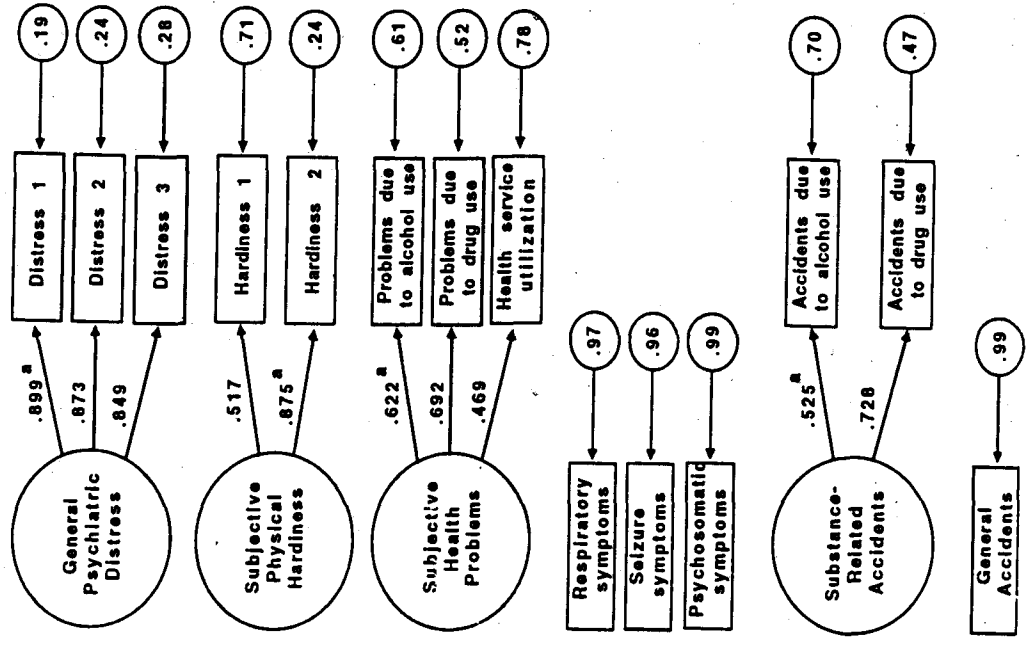


STEIN, SMITH, GUY, & NEUTLER
APPLIED PSYCH. (1993)

Figure 1



Time 2



GUY, SMITH, & BEUTLER, 1993 Psychology & Health

Figure 2

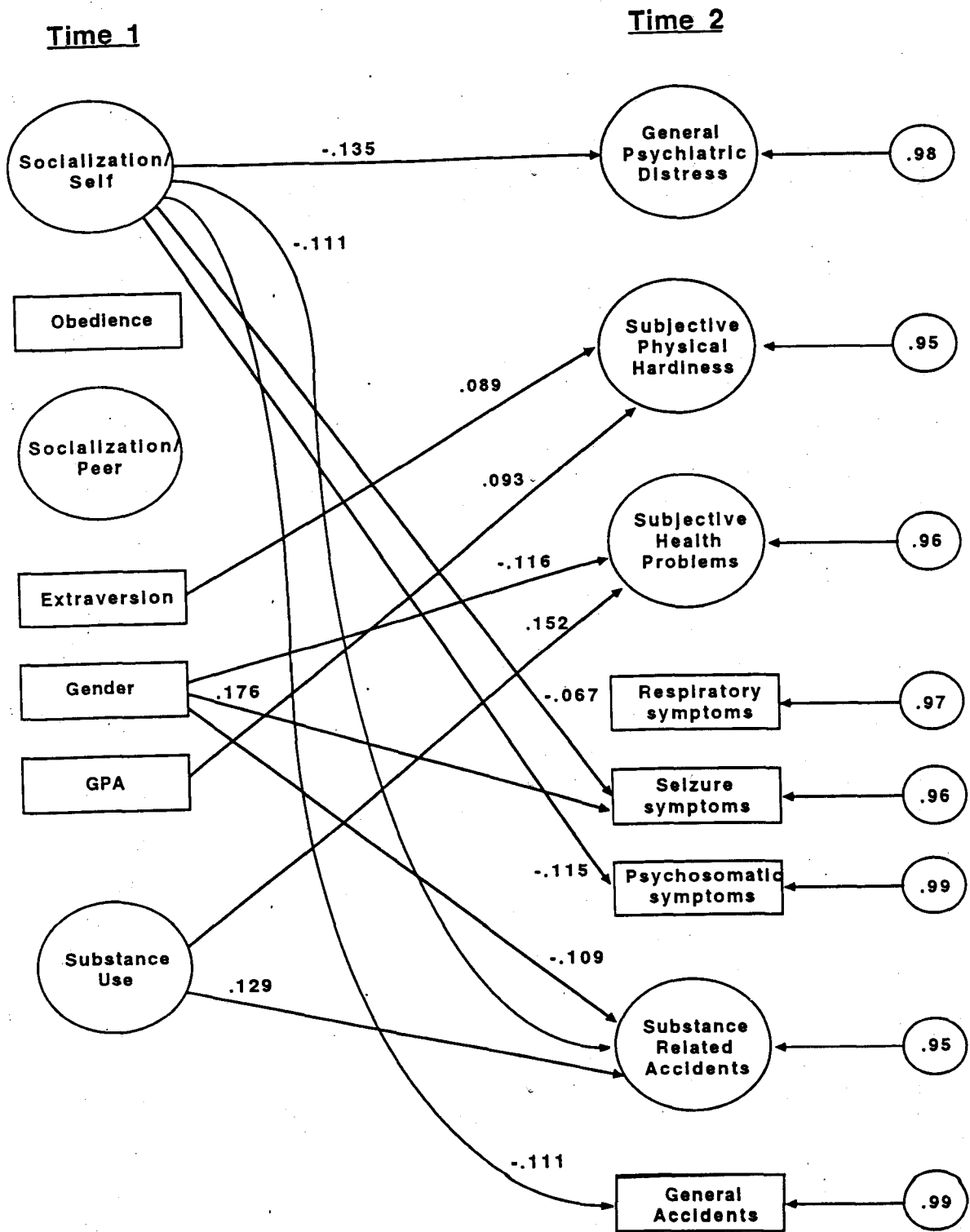


Figure 3

Time 1

Time 2

